Optimal Design and Manufacture of Thin-Walled Polystyrene Structures

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In this study, we investigated the implementation of an automatic procedure for optimizing thermoformed thin-walled structures. Such objects are created in great numbers, especially in the food packaging industry. The methodology for the optimal design of such structures is based on the use of a parameterized geometry model created within an interactive design environment. By varying the parameters associated with the computer-aided design (CAD) model, one can create a rich variety of possible designs. One can then subject these designs to physical analysis to calculate their physical properties, and thus select an optimal design. The two distinct stages of this process—the prediction of the shape of the thermoformed structure, and the physical behavior of the structure—were validated by experiments. This article reports the experimental investigation of the deformation behavior of polystyrene, the mechanical behavior of specially prepared deformed polystyrene sheets, and the response to loading of a hemispherical structure (used in the validation). POLYM. ENG. SCI., 45: 694–703, 2005. © 2005 Society of Plastics Engineers

INTRODUCTION

This article stems from a collaboration between the Polymer Group in the Department of Physics and Astronomy, and the School of Mathematics at Leeds University. It describes the experimental work undertaken to enable the validation of a computer-aided procedure for the design and functional optimization of thermoformed thin-walled structures made from polymers. Such structures include containers for dairy products such as margarine, ice cream, and yogurt; trays for preprepared meals; and clear packs for sandwiches, wine, and beer.

In previous publications, Ugail et al. [1, 2] presented an efficiently parameterized computer-aided design (CAD) model. This model uses the partial differential equation (PDE) method [3] for designing complex shapes. This method adopts a boundary-value approach in which the shape of the object in question is decomposed into a series of surface patches bounded by “character-lines,” whereby the number of patches is kept as low as possible. Using the boundary data appropriately defined along the character-lines, the PDE method generates smooth surfaces between them by solving an appropriately chosen partial differential equation. An important feature of the method is that the shape of the surface produced is efficiently parameterized with a minimal number of shape parameters, thus enabling automatic optimization to be carried out.

The rationale for the experimental work described in this article is to provide information regarding material properties for the computer-aided procedure. The procedure for achieving this is described in detail elsewhere [4], and involves the automatic optimization of the mold shape, taking into account the strength of the final object and its thickness distribution, thus reducing the need to perform inefficient and expensive “trial and error” experimentation using physical prototypes. The PDE method for parameterizing geometry is utilized, enabling the creation of a wide variety of possible mold shapes on which appropriate analysis can be performed. The analysis involved is a finite-element calculation of the behavior of the containers when they are externally loaded, for which knowledge of the material properties (as determined by the experimental program described below) is required.

There are thus two requirements for the data on polymer material properties: First, it is necessary to be able to predict the plastic deformation in the polymer that occurs as a result of the initial forming operation to make a container of defined geometry. Second, it is necessary to define the
Design Strategy

Figure 1 shows schematically the design strategy adopted here for the automatic design of thermoformed thin-walled structures made from polymers. The particular focus was on the design of structures used for food packaging, and the design objective was to minimize the amount of polymer used, subject to various functional constraints on the strength and volume of the packaging.

An initial design for the mold geometry is created using a parameterized CAD model produced by the design methods described above. From this geometry the relevant physical characteristics of the corresponding container are calculated using finite-element analysis, and the value of an “objective” function—which is a measure of the feature of the design that one wishes to optimize—is obtained. By making use of a method for numerical optimization (based in this case on the standard Broyden-Fletcher-Goldfarb-Shanno method), automatic variations in the parameters associated with the shape of the CAD model are carried out in order to determine whether or not the shape is optimal. If it is, the design process stops, but if not, the optimization method produces a new design with an improved value of the objective function. The physical characteristics of the new shape are calculated using finite-element analysis as before, and the whole process is repeated until an optimal design is found.

For the types of food containers that were considered here, one of the relevant design objectives was to design a container that possessed the minimum amount of material but met certain strength requirements. For example, the problem of stacking yogurt pots on top of each other, for the purposes of transportation or display on supermarket shelves, was dealt with. A pot at the bottom of a stack will experience a loading due to the weight of the pots above it. In these circumstances, excessive shear stress in the walls of the pot can cause damage to the pot. At the same time, because of environmental concerns and to reduce material costs, it is desirable to keep the amount of polymer used as low as possible. Thus a suitable design objective in this case might be to reduce the amount of plastic in the pot subject to maintaining a certain degree of strength in the walls. The strength of the walls might be characterized by calculating the maximum shear stress that occurs in the pot for a given applied load. In this work the shear stress was calculated by means of thin-shell finite element analysis.

To perform the appropriate physical analysis one must simulate the thermoforming process to calculate the thickness distribution of the container walls. For this purpose we made use of a commercial software package [5] called T-SIM (Accuform 2000), which employs the K-BKZ model to simulate the forming process. In particular, given the mold shape and initial thickness of the polymer sheet prior to forming, as well as other relevant parameters for the forming process (e.g., temperature), T-SIM is able to predict the thickness distribution of the walls of the container post forming.

The particular nonlinear time-dependent K-BKZ model used here involved the single integral approach proposed by Kaye [6] and Bernstein et al. [7]. The time integral constitutive equation is

\[ \tilde{\sigma}(t) = \frac{1}{t} \int_{0}^{t} \mu(t - t') h(I_1, I_2) \mathbf{B}(t, t') dt' \]  

(1)

where \( \tilde{\sigma}(t) \) is the stress tensor, \( \mu(t - t') \) is a time memory function, \( h(I_1, I_2) \) is a damping function of the two strain invariants \( I_1 \) and \( I_2 \), and \( \mathbf{B}(t, t') \) is the finger strain tensor. The time memory function can be expressed in terms of the relaxation moduli \( \alpha_i \) and relaxation times \( \tau_i \) according to

\[ \mu(t - t') = \sum_{i=1}^{N} \left[ \frac{\alpha_i}{\tau_i} e^{-(t-t')/\tau_i} \right] \]  

(2)

and the damping function was a one parametric Wagner type [8, 9] of the form

\[ h(I_1, I_2) = \frac{1}{1 + \alpha \sqrt{(I_1 - 3)(I_2 - 3)}} \]  

(3)

Temperature effects are included via a WLF temperature dependence of the material parameters [10]. The T-SIM thermoforming software uses a parameterized form of the model generated from a second related commercial package, T-Simfit, and produced by curve-fitting to experimentally obtained processing data.

The first major purpose of the experimental program
Experimental Strategy

To be able to model the behavior of processed shells it is necessary to define the geometry of the thermoformed product and the local mechanical properties of each element. It is difficult to produce directly equivalent control samples in the laboratory because industrially produced shells are often obtained at strain rates that are much higher than those achievable in the laboratory and under nonthermal equilibrium conditions. The strategy adopted here was to make experimental measurements under controlled conditions of fixed temperature and strain rate, and to apply these measurements to the industrial circumstances using two fundamental principles that have been applied successfully elsewhere [11]. The first principle assumes that the flow stress of polystyrene at a particular temperature relates only to the current strain rate and total plastic strain. The second related principle assumes that the mechanical properties relate uniquely to the deformation ratios imposed, i.e., to the total plastic strain.

These principles are limited in the sense that they bypass the relationship of physical properties to molecular characteristics. Generally, the mechanical properties of oriented polymers are known to depend on the type and degree of orientation. For noncrystalline polymers, such as polystyrene, the molecular anisotropy developed during processing depends on the competition between the orienting effects produced by deformation of the topological entanglements and the relaxation effects that arise from processes such as reptation. The different relaxation modes have characteristic times so that factors such as the strain rate, processing temperature, and cooling conditions can all have an influence on the molecular orientation frozen in the processed item. Nevertheless, these principles enable such issues to be successfully bypassed when they are applied carefully.

Material

A 1:1 by weight blend of general purpose polystyrene (Crystal, GPS FinaAto 1540) and high impact polystyrene (HIPS FinaAto 7240) was the material under examination. Table 1 lists some information, published by the manufacturer, for these two grades. The polystyrene blend was supplied in the form of a melt-extruded sheet (0.8 mm thick).

Measurements

Three distinct sets of experimental measurements were performed in the course of this work. First, the processing behavior was investigated with the intention of predicting the shapes that would be developed in thermoforming. Second, the mechanical properties of oriented samples were obtained for use in finite element software designed to predict the response of a processed hemispherical shell. Finally, measurements were performed on hemispherical processed shells for comparison with predicted behavior.

Uniaxial Deformation

Detailed measurements of flow stress during uniaxial deformation were made on an Instron tensile testing machine operating in true strain rate mode. Dumbbell-shaped samples (width = 18 mm, gauge length = 54 mm) were cut from processed sheets and then mounted vertically in the grips of the tensile testing machine in which the upper grip was attached to a load cell mounted in the crosshead, which moved vertically upwards. The lower grip remained fixed throughout the test. In true strain rate mode, the crosshead speed at time $t$ after the start of the test is related to the grip separation at time $t$ according to the equation

$$v(t) = \frac{l(t)}{l_0}$$

where $v_0$ and $l_0$ are the initial speed and grip separation, respectively. For homogeneous deformation of the sample

<table>
<thead>
<tr>
<th>Grade</th>
<th>MFI (g/10 min)</th>
<th>$M_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPS FinaAto 1540</td>
<td>12</td>
<td>210,000</td>
</tr>
<tr>
<td>HIPS FinaAto 7240</td>
<td>4.5</td>
<td>180,000</td>
</tr>
</tbody>
</table>
between the grips, this equates to the constant strain rate 
\[ \dot{\varepsilon} = \frac{v_o}{l_o} = \frac{v(t)}{l(t)}. \]

Prior to deformation, each sample was marked at five positions along its length with transverse lines. By following the local strains as measured by the separation of different pairs of these lines, a video extensometer was used to confirm that strain in the material was homogeneous during deformation and consistent with that predicted from the programmed displacement of the crosshead. Measurements were performed at temperatures between 110°C and 130°C, and at strain rates in the range of 0.05–5 min⁻¹.

The drawing loads were generally very low, and reasonable accuracy could only be obtained with a very sensitive load cell. A 10 N full-scale deflection cell was utilized, and care was taken to minimize offset errors and to identify any drift during the course of the experiment. The temperature enclosure had a thermal gradient of about 2°C over the chamber length (from bottom to top) so that the sample moved into a slightly warmer zone as drawing progressed. However, the temperature gradient over the length of sample viewed by the video extensometer was less than 1°C, and the drawing temperature recorded was equivalent to this region.

The procedure used to calculate the flow stress was as follows: The load and the video extensometer strains were recorded as a function of time for uniaxial tensile deformation of the samples using the constant strain rate mode for a series of chosen strain rates. The video extensometer measurements were used to confirm that the strain was homogeneous, and to find the true constant strain rate. Typically, the strain rate in the sample gauge length was 2.5–4% greater than the imposed strain rate. The corrected strain rate in the gauge length was used to calculate the local cross-sectional area as a function of time (on the assumption of constant volume deformation) and thereby the true stress. For drawing at 130°C, the deformation in the gauge length was slightly inhomogeneous, possibly because of the temperature gradient in the oven, but the above procedure could still be applied to find the stress strain curves for two different but closely similar strain rates, and an average of the two was taken.

Detailed checks on homogeneity and the true local strain rate could only be performed up to a draw ratio of about 5. At higher draw ratios the lines toward the upper part of the sample moved out of the field of view of the video extensometer, or line definition became too indistinct. The flow stress at draw ratios higher than 5 were obtained assuming that the constant strain rate recorded at low deformation was still applicable.

**Constant-Width Deformation**

Some limited measurements of the flow stress were obtained for constant-width deformation using the T M Long biaxial stretcher. A significant advantage of this machine is its gripping mechanism, which permits successful deforma-

tion of samples over a wide range of conditions. The main disadvantage is that deformation is performed at a constant speed rather than a constant strain rate. We cut 60-mm-square samples from an 0.8-mm-thick isotropic sheet and mounted them in the pneumatically controlled clamping system. Sets of six grips along each of the four sides gripped the sample, leaving about 5 cm of unclamped sample between opposing sets of grips.

**Mechanical Properties**

Mechanical properties were measured on a series of samples cut from oriented sheets, prepared by drawing to fixed stretch ratios. These large oriented sheets were obtained from a 0.8-mm-thick isotropic sheet by drawing, on a custom-built [12] biaxial drawing machine, 18-cm square samples cut from the melt-extruded roll. Two deformation modes—uniaxial and constant width—were employed. In uniaxial mode, the 18-cm square squares were mounted in two sets of grips, leaving a gauge length of about 12 cm, while the sides of the sheet were left unclamped. In constant-width drawing, the sides were clamped and these clamps remained at a fixed transverse separation during drawing so that the sample width was maintained at about 18 cm. A true constant shear strain rate of 0.02 s⁻¹ was used in both modes. Before drawing, the sheets were marked with a square grid so that areas of homogeneity could be selected from the drawn sheets. Constant-width drawing yielded homogeneous samples with a width of 12–14 cm, whereas uniaxial deformation produced homogeneous samples with a width of typically 8 cm, although this depended on the imposed axial strain. Deformation was performed in an environmental chamber with the temperature controlled to ±2°C.

We determined the elastic constants of the oriented and isotropic material using a range of techniques, because no single technique was suitable for measuring all the constants.

**Ultrasonic Measurements**

An ultrasonic immersion technique using custom-built equipment [13] was used to measure the elastic properties of the isotropic material. This technique is most suitable for measuring the complete set of elastic constants in samples that are at least 4–5 mm thick, but it can also be used for thinner samples (although the results are less accurate). Basically, when an ultrasonic pulse is incident on the sample, a shear wave and a longitudinal wave can be propagated, the velocities of which depend on the elastic constants of the material in the plane in which the two waves propagate. Careful measurement of the time of flight for both waves in the sample for different angles of incidence for the ultrasonic pulse (achieved by rotation of the sample about an appropriate axis) enables the four elastic constants for the plane perpendicular to the axis of rotation to be determined.
The samples examined all had a square profile of approximately 5 cm, but varied in thickness. The thinnest sample was cut from the nominally isotropic melt-extruded sheet and had a thickness of 0.8 mm. Measurements were also made on a stack of four of these sections, and a third sample (3.7 mm thick) was obtained by molding a stack of six sections of the extruded sheet at 160°C. This final sample was prepared for examination to avoid complications introduced by internal reflection at the interfaces in the unmolded stack.

**Dynamic Mechanical Measurements**

The dynamic mechanical behavior was examined with the use of a Rheometrics RSA 2. We tested the samples in uniaxial tension by applying an oscillatory axial strain of 0.02% amplitude and measuring the force developed in the sample. The sample was maintained under a constant preload of 2N, which produced a uniaxial strain of approximately 0.06% at −40°C. The precise value of prestrain depended on the modulus, which varied with temperature, but for the range of −120°C to 80°C the modulus change was sufficiently small so that the prestrain remained at (0.06 ± 0.015)% This level of prestrain ensured that the sample remained in tension through the complete cycle. The rectangular cross-section of the sample was typically 3.5 mm × 0.25 mm, and the length was 35 mm.

**Ten-Second Isochronal Creep Measurements**

The 10-sec isochronal creep modulus was measured with dead-load creep equipment (described in detail in Ref. 14). Basically, the sample is mounted between two clamps and subjected to a uniaxial tensile stress. The sample extension after 10 sec is measured accurately (using a LVDT) for a range of applied stresses. The stress strain curve is constructed from these data, and the gradient at a strain of 0.1% is calculated. For the samples investigated here (in the range of up to 0.12% strain), the stress strain curves were linear and there was no detectable time dependence, indicating that the material behaves elastically.

**Modulus and Poisson’s Ratio Measurements Using a Video Extensometer**

A Messphysik video extensometer was used to monitor the axial and transverse strains of samples subjected to an imposed uniaxial deformation by an Instron tensile testing machine. Measurements were performed at a constant crosshead speed (equivalent to an initial axial strain rate of 1.39 × 10^-4 s^-1) and a temperature of 23°C. Typically, the samples had a gauge length of 9 cm and a width of 6 mm, and were cut from oriented sheets with a diamond saw and masking tape to minimize the generation of flaws and microcracks. The drawn sheets were wide enough to allow samples of adequate length to be cut at inclinations of 45° and 90° to the draw direction so that the anisotropy in the elastic properties could be measured. A rectangular shape (approximately 10 mm × 5 mm, drawn in black ink) acted as the target used by the video extensometer system when the axial and transverse strains were measured. In all cases examined, the axial strain measured by the video extensometer agreed well with that determined from the clamp separation, indicating that the strain was homogeneous.

We obtained the Poisson’s ratio by measuring the gradient of the strain vs. time curves for both axial and transverse strains between the 0.2% and 0.6% strain levels and taking the ratio of these gradients. Typically, the error that arises from multiple tests on the same sample is ±0.03.

**Hemispherical Shells**

Thermoformed hemispherical shells with a basal plane outer diameter of 87.8 mm (94 mm including the grip rim) were produced from a 0.78-mm-thick isotropic sheet at nominal processing temperatures of 80°C, 114°C, 130°C, and 163°C. The process was performed on custom-built thermoforming equipment at the Huhtamaki Laboratory in Leeds. Prior to deformation, square grids were printed on the sheet so that the local deformation induced by thermoforming could be measured. Before thermoforming, the sheets were preheated under infrared radiant heaters for about 10 sec (depending on the desired temperature and heater setting) and then transferred to the thermoforming station. The preheated sheets were then located above the room temperature mold and the clamping system, which held the rim, was brought into position. A thermoforming pressure of 50 psi, activated by a solenoid-controlled valve, was introduced on the exterior surface of the sheet and maintained for about 4 sec, forcing the sheet into the mold cavity. After this time, the pressure was released and the clamping system was withdrawn a second or so later so that the thermoform could be removed from the mold.

Compression tests were performed on the fabricated shells by means of an Instron tensile testing machine, operating at a constant crosshead speed of 0.5 mm min^-1 and room temperature (21°C). The samples were mounted between two heavy steel plates with flat surfaces parallel to one another and perpendicular to the axis of compression.

We measured the height of each hemisphere during compression tests using a reference position of zero when the two compression plates were in contact. The heights were therefore measured from the lower side of the basal plane. With the exception of shells processed at 163°C (44.6 mm), the heights were independent of the processing temperature (all being 44.5 mm). The difference between the shells processed at 163°C and the other shells was very small but measurable.

The thickness at positions along a circumferential arc passing through the pole of the hemisphere was measured for hemispheres processed at 130°C and 163°C. Results are presented in Fig. 2 for a hemispherical shell processed at 130°C. Measurements for the hemisphere processed at 163°C agreed with those of the hemisphere processed at
130°C within experimental error. The only way we could distinguish between hemispheres processed at those two temperatures was to measure the height above the basal plane, as mentioned above. Also shown in Fig. 2 are the predicted results obtained using the T-SIM thermoforming software and measurement of the flow behavior of polystyrene. There is a small discrepancy between experiment and theory in the walls of the hemisphere, but generally the agreement is quite good.

RESULTS

Processing Behavior

Many detailed measurements of the processing behavior were obtained in uniaxial deformation, but only two representative graphs are shown here. Figure 3 shows the development of the true drawing stress as a function of strain rate at different levels of strain for a drawing temperature of 120°C. The strain rate sensitivity and strain hardening are both evident. Figure 4 shows, for three different temperatures, the strain rate dependence of the drawing stress at a constant strain comparable to the maximum imparted during the processing of the thin-walled structures of interest here. However, it should be pointed out that the industrial processing of the polystyrene containers takes place at about 140°C and a strain rate of 5 s⁻¹, and these conditions were beyond the range of the current experiments.

The detailed measurements in uniaxial deformation were used by the T-Simfit software to produce the material profile accessed by the T-SIM thermoforming software when the shape generated by specific processing conditions was predicted. A comparison between the fit and the experimental data is shown in Fig. 5. It can be seen that the model gives very good agreement with the data at 110°C after yield. The agreement at higher temperatures is not quite so good, indicating that the temperature dependence of the parameters is not well characterized. Nevertheless, provided that the specific processing conditions used in the thermoforming are not far removed from those used to measure the flow stress behavior, the model seems to be adequate.

We also examined two other material models; however, since their usefulness in the present context is limited because they cannot be modeled by the T-SIM software for analyses of thermoforming behavior, only brief details are given here. One of these models was a constitutive model based on the ideas of Ogden [15]. The results of the curve-fitting T-Simfit software are in excellent agreement with this model, and it is probable that later versions of T-SIM will allow operators to use this material model, offering the prospect of even better predictions.

We also examined the hyperelastic deformation model of Edwards and Vilgis [16] in an attempt to introduce a physical understanding to the deformation behavior. This model, developed from theories of a deforming network, expresses
the flow stress in terms of four parameters: the cross-link and slip-link densities, the mobility of the slip-links, and the network inextensibility. Fitting was performed with the constraint that the cross-link density was zero. Generally, the quality of the fit to the experimental data was reasonable, but the three other parameters showed a dependence on both temperature and strain rate, implying that temperature and strain-rate effects are not easily separable.

There is a question concerning the validity of using data obtained in uniaxial tensile deformation to predict behavior under alternative modes of deformation. Sweeney and Ward [12] showed that in PVC above the glass transition temperature, the different modes of uniaxial, planar, and equibiaxial tensile deformation are comparable provided the results are expressed in terms of the octahedral shear stress and strains. Attempts to confirm that this approach is applicable to polystyrene encountered some difficulties because at temperatures above 100°C, the drawing loads in constant-width drawing were too small to measure on the existing equipment. Figure 6 shows the processing behavior for constant-width drawing at a temperature of 100°C and a constant crosshead speed of 7.5 mm s⁻¹. Also shown in this figure is the predicted behavior based on measurements from uniaxial deformation at a strain rate of 5.2 s⁻¹ and assuming an Edwards-Vilgis hyperelastic deformation model [16]. The constant-width deformation was performed at constant speed, not constant strain rate, and the comparisons between the two different modes were based on equivalent average octahedral strain rates. Hence, for constant-width drawing, at low strains the strain rate would be greater than this average, while at high strains it would be less than the average. This would lead to a predicted underestimate of the shear stress at low strain and an overestimate at high strain. This is consistent with Fig. 6, supporting the use of the uniaxial deformation behavior as a basis for predicting more general deformation behavior, at least for the conditions considered here.

The main practical finding from analysis of the processing behavior is that the T-SIM software can predict quite well the shape of thermoformed polystyrene hemispherical shells. The use of a K-BKZ constitutive model to represent the material was successful, although there were some clear limitations. It was shown that a better fit could be obtained by use of the Ogden model, but this is purely empirical and does not include rate effects.

**Mechanical Properties**

Figure 7 shows results obtained from 10-sec isochronal creep measurements on samples of polystyrene drawn under nominally uniaxial conditions to different draw ratios. The majority of the results relate to samples drawn at 100°C, and although there is a slight increase in modulus, especially at...
a low draw ratio, generally the drawing process produced very little enhancement of the modulus. This small change in modulus is even less pronounced when deformation is performed at higher temperatures.

The anisotropy of Young’s modulus and Poisson’s ratio in oriented sheets of polystyrene, obtained by constant-width drawing at 100°C, is considered in Figs. 8 and 9. $E_0$, $E_{45}$, and $E_{90}$ denote the Young’s modulus measured in the direction inclined at 0°, 45°, and 90°, respectively, to the primary draw direction. The modulus results were obtained from the gradient of the stress strain curve between 0.2% and 0.4% macroscopic strains using the video extensometer. Generally, $E_0$ shows a small increase with draw ratio while values for $E_{45}$ and $E_{90}$ are relatively unchanged, although there appears to be an initial small drop at low draw ratios in the latter case. In contrast, the Poisson’s ratio appears relatively constant for inclinations of 0° but drops slightly for inclinations of 45° and 90°. The data in Figs. 8 and 9 can be combined to calculate the shear modulus on the basis that

$$
\frac{1}{E_{45}} = \frac{1}{4} \left[ \frac{1}{E_0} + \frac{1}{E_{90}} - 2 \frac{\nu}{E_0} + \frac{1}{G} \right]
$$

where $\nu$ is the Poisson’s ratio, and $G$ is the shear modulus. The results in Fig. 10 show that there is no detectable change in the shear modulus with the draw ratio.

Table 2 shows the values of the extensional modulus and shear modulus found using the different techniques. The ultrasonic technique, which relates to measurements made at very small strains, was performed at 2.25 MHz. The 10-sec technique relates to the initial 0.1% strain region and is derived by measuring the final strain after the stress has been applied for 10 sec. In the Instron technique a macroscopic strain of 0.833% was developed uniformly in 1 min, and the modulus was found from the 0.2–0.4% strain region. Finally, the dynamic technique relates to an imposed strain of amplitude 0.02% applied at a frequency of 1 Hz.

The results summarized in Table 2 show very good agreement between values of the extensional modulus obtained from the different techniques. For the relatively low-
frequency techniques, the differences are consistent with an observed small nonlinearity in the elastic region of the stress strain curve, and with the presumed small dependence on frequency suggested by the dynamic mechanical measurements. It is only the ultrasonic measurement that differs significantly, but this is not totally unreasonable considering that it relates to a significantly higher testing frequency. The same explanation can account for the difference between the ultrasonic shear modulus and the value predicted by applying the anisotropy equation to low-frequency measurements.

The general conclusion derived from these tests is that, to a first approximation, at low strains the material is linearly elastic, with the elastic constants showing very little dependence on the draw ratio in the range tested (up to a draw ratio of 3.3). At a finer detail, there is a degree of nonlinearity in the elastic behavior, with a 10% drop in the modulus being observed as the macroscopic strain is increased to 1%. Similarly, there is evidence of viscoelastic behavior in the frequency dependence of the modulus. The development of the extensional modulus with the draw ratio may be as much as 20% for higher draw ratios, but is considerably less for the range of draw ratios (typically a maximum of 2.5) encountered in the processed shells.

**Hemispherical Shells**

**Deformation Behavior.** The behavior of the hemispheres under compressive loading is shown in Fig. 11 for hemispheres processed at 130°C and 163°C. Multiple tests were performed on each hemisphere up to a displacement of 2 mm. At the end of each test the crosshead was returned rapidly to its initial position and a time interval of 10 min was allowed for sample recovery before the subsequent test was performed. Hemispheres subjected to multiple tests in this manner showed no indication of visual damage after the tests. However, the load extension curves at extensions beyond a threshold of about 0.8 mm are not superimposable. The load at any given extension (above this threshold) in tests subsequent to the initial test was slightly but consistently lower. At lower extensions, below the threshold, the load is superimposable, which suggests that the behavior in this region is genuinely elastic.

Figure 11 shows that although there is a detectable difference in the response to loading at room temperature for the hemispheres processed at 130°C and 163°C, there are considerable similarities. (The behavior of hemispheres processed at lower temperatures is the same as that of the hemisphere processed at 130°C.) Both show an initial linear response, followed by a region where the gradient increases before it decreases again at displacements greater than 0.7 mm. Also shown in Fig. 11 are the results predicted by the finite element package for a hemisphere processed at 130°C assuming that polystyrene is a linearly isotropic solid. (The predicted results do take into account the variable thickness of the processed shell.) This approach was adopted in the first instance because it is simple to implement and is supported by the experimental measurements of physical properties for polystyrene at the levels of strain encountered in the hemispherical shells. Clearly, the main features are

**FIG. 11.** Load extension curves for polystyrene hemispheres tested in compression at room temperature. Hemispheres processed at temperatures ●, 130°C; ■, 163°C; and ——, predicted for a hemisphere processed at 130°C.

<table>
<thead>
<tr>
<th>Method</th>
<th>Extensional modulus/GPa</th>
<th>Shear modulus/GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ultrasonic</td>
<td>10 sec</td>
</tr>
<tr>
<td>Sample isotropic</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Uniaxial draw ratio</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.52</td>
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</tr>
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<td>1.87</td>
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<td>2.37</td>
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<td>3.25</td>
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<td>Constant width, draw ratio</td>
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<tr>
<td>3</td>
<td>2.73(2)</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2.** Comparison of extensional and shear moduli from different techniques.
well replicated using this approach, and it is likely that agreement would be improved by accounting for the small degree of anisotropy observed in the physical properties and by a better modeling of the hemisphere profile considered in Fig. 2.

CONCLUSIONS

Experimental measurements of the drawing behavior of isotropic polystyrene and the mechanical behavior of oriented sheets, prepared by drawing to fixed draw ratios, were used to help validate a CAD package designed to produce an optimal shape for a thermoformed polystyrene structure subject to certain constraints. We have shown that by using the experimental data reported here, the software can predict the behavior of a chosen structure (in this case a hemispherical shell) quite well, offering the real prospect of assessing many different possible structures without having to manufacture and test each structure individually. The approach was simplified by the fact that for the range of deformation encountered in typical thermoformed structures, polystyrene behaves like an isotropic linear elastic solid at low strains. Improvements in the ability of the software to predict the behavior of thermoformed structures are likely to follow from a better model than the K-BKZ model used here, in particular for the processing behavior and development of finite-element analysis software to cope with anisotropic materials.

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